Constrained Linear Quadratic Optimal Controller for Motion Control of ATMOS Driving Simulator

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Abstract - *In this paper, a new model based motion cueing approach based on a constrained finite horizon linear quadratic optimal controller for ATMOS driving simulator is presented. In addition, a new approach is used to represent the sustained acceleration for the tilt coordination. The basic idea of the new approach is based on minimizing the difference between the perceived motions of the vehicle and the simulator. Compared to the conventional motion cueing algorithms, the proposed approach provides a more realistic impression, the workspace of the simulator is better exploited and all the constraints of the driving simulator are respected. Models of the human perception system in combination with simulator dynamic model are used to design the constrained linear quadratic optimal controller. The proposed controller has a feedback from the measured accelerations of the simulator and has a feedforward part from the reference trajectory.*

Keywords: *Driving simulators, Motion Cueing, Optimal control, Perception model, Online optimization.*

Introduction

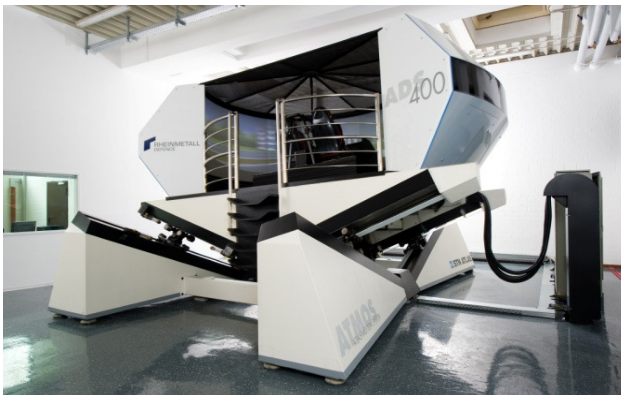

Figure 1: ATMOS Driving Simulator

Driving simulators are used in many different areas. They can be used for research purposes to study the behavior of the driver or develop and evaluate the new subsystems of the vehicle, or they can be used for training of drivers or for entertainment as in video games.

The general goal of any simulator is to give a realistic impression of the vehicle motion to the driver. Due to the limitations of workspace and the technological constraints of the motion systems, the vehicle translational accelerations and angular velocities generated by the vehicle dynamic model cannot be provided directly to the motion systems. Therefore, vehicle signals should be reproduced in a specific manner in order to produce admissible motion commands that could provide the simulator's driver in virtual reality environment the same feeling as in reality. The method that is used to reproduce the vehicle accelerations and velocities is commonly known as motion cueing algorithm.

One of the most widely used motion control strategies used for motion cueing is called classical motion cueing algorithm (figure 2). It was initially developed for the flight simulators [Sch21, Gra10]. This strategy is a combination of different linear filters used to render the vehicle signals by extracting a specific bandwidth from the vehicle signals. In this algorithm, the high pass filters are used to extract the transient part from the vehicle signals. Then, the extracted signals are single or double integrated to find the desired positions or angles. Since the workspace of the motion system is limited, the representation of the sustained component of the longitudinal and lateral vehicle accelerations is executed by tilting the moving platform. This mechanism is known as tilt coordination. The last stage of the classical motion cueing is the washout process which is required to bring the simulator back to its neutral position. The washout and the tilt coordination should be carried out under the perception thresholds of the driver [Rey20].

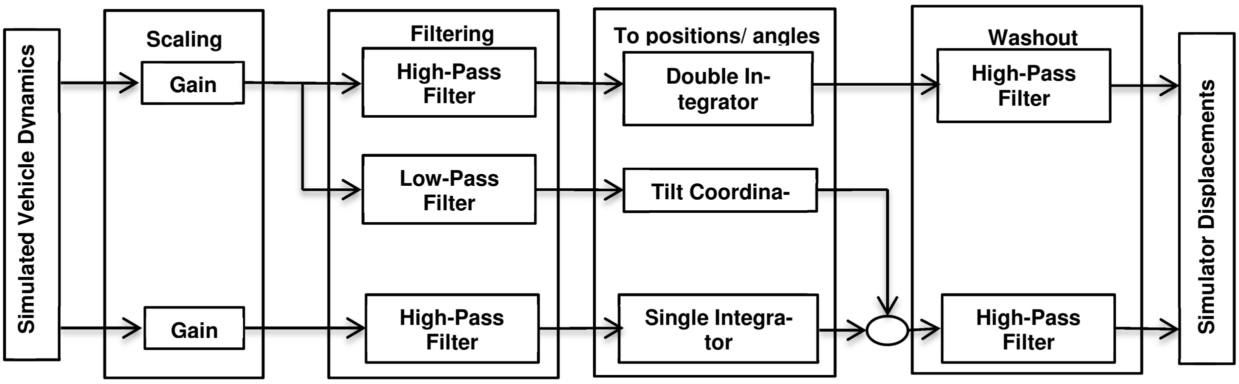

Figure 2: Classical Motion Cueing

Different algorithms based on the classical motion cueing algorithm have been developed to render the vehicle signals such as adaptive motion cueing where the parameters of the filters are changeable and computed at each time step of simulation [Rei18, Rei19, and Par17], and the optimal algorithm which uses high order filters chosen through an optimization method consisting of the human perception model in order to reduce the perception error between the driver at the vehicle and the driver at the simulator [Rei18, Rei19, Tel26, and Siv24].

The majority of developed approaches are based on extracting the transient and the sustained cues from the vehicle accelerations using linear high- and low-pass filters respectively [Neh15, Neh16]. The parameters of these filters (cutoff frequency, damping, and gain) are adjusted according to the worst case and tuned traditionally by trial and error heuristic procedures, therefore the constraints of the simulator are not always maintained and the workspace is not exploited very well. In addition, few model-based approaches have been developed for motion control of the driving simulators but they have some limitations [Dag6 and Aug1].

In this work, a completely different motion cueing approach based on the constrained linear quadratic optimal controller will be presented. The design of the linear quadratic optimal controller has been extensively discussed in the literature, see for example [Bri4, Chm5, and Sco22] and the idea of integrating the constraints in the controller has also been discussed in [Goo9].

Constrained linear Quadratic Optimal Controller

A. Driving Simulator Dynamics and Controller

The ATMOS driving simulator (figure 1) which is used at the Heinz Nixdorf Institute in University of Paderborn for research purposes has a projection system with 270° to view the details of the environments during the experiments and it is constructed from two dynamical parts with 5 degrees of freedom (DOFs). These two parts are independent of each other and the system is fully actuated. Therefore, each degree of freedom can be controlled independently. The first dynamical part is the moving platform (figure 3). It has 2 DOFs and is used to simulate the lateral and longitudinal accelerations of the vehicle. It can move in the lateral plane and at the same time it has the ability to tilt around lateral axis with a maximum angle of 13.5° and around the longitudinal axis with a maximum angle of 10°. Four linear actuators are used to control the movements in both directions. However, the control of each two actuators in each direction is done independently but synchronously. The second dynamical part is the shaker system (figure 4) which has 3 DOFs to simulate the roll and pitch angular velocities and the vertical acceleration of the vehicle. It is driven by three drive crank mechanism (three actuators).

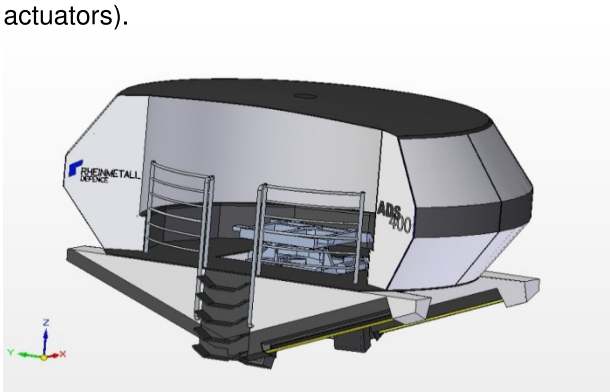

Figure 3: Moving platform with cockpit

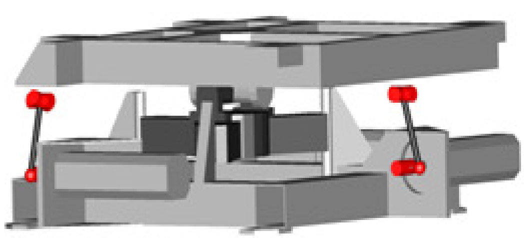

Figure 4: Shaker system

The two dynamical parts of the simulator are modeled independently and the principle of Newton-Euler is used to model the accelerations of the two dynamical parts in minimal coordinates. The dynamics of the moving platform and the shaker mechanical system can be written as:

$$M_q(q)\ddot{q} + h_q(q,\dot{q}) = Q_q(t) \quad (1)$$

where $\ddot{q}, \dot{q}, q$ represent the accelerations, velocities and positions(angles) respectively. $M_q(q)$ is the inertial matrix, $h_q(q,\dot{q})$ represents the gravitational and centrifugal forces and $Q_q(t)$ is the external forces and torques acting on the simulator during its motion.

To synthesize a controller for actuators of motion system, a controller based on the inverse dynamic technique which is known as computed torque method, is used to handle the high nonlinearities of the dynamics of the simulator. The computed torque controller generates the input command:

$$Q_q(t) = M_q(q)v + h_q(q,\dot{q}) \quad (2)$$

where v is the new control input to be designed. The typical choice of v is:

$$v = \ddot{q}_d + k_v(\dot{q}_d - \dot{q}) + k_p(q_d - q) \quad (3)$$

where q_d is the desired position. k_v and k_p are the gain matrices. It follows that the resulting linear error dynamics are:

$$\ddot{e}_q + k_v\dot{e}_q + k_p e_q = 0 \quad (4)$$

The error dynamics are exponentially stable by a suitable choice of the gain matrices.

It is important to note that this kind of control method converts the complicated nonlinear controller design into a simple design problem for linear system [Sic23]. By using this principle, the whole dynamic system can be assumed to be a linear and decoupled system. The following linear system represents the dynamics of the motion system for one degree of freedom

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m u_m(t) \\ y_m(t) &= C_m x_m(t) + D_m u_m(t) \end{aligned} \quad (5)$$

where $u_m(t)$ represents the acceleration control inputs of the motion system, $x_m(t)$ is the current state vector, $y_m(t)$ is the acceleration of the motion system, and A_m, B_m, C_m, D_m are matrices of proper dimensions.

B. Motion Perception System Dynamic Modeling

Human beings can detect the movements by the vestibular system which is located in the inner ear and it plays the main role to provide the perceptual system of the human beings with information about linear and angular inertial movements of the body. The vestibular system consists of two sensory parts, the semicircular canals that detect the angular motion and the otoliths organs that are sensitive to the translational motion and gravity i.e. specific force.

The biological phenomena which reflects the response of the vestibular system has been deeply studied [Fer7, Man13] and several dynamical models for the vestibular system based on empirical tests as well as describing it's working principle are available in the literature.

Young and Meiry [Mei14] modified the second order low pass filter otoliths model proposed by Meiry [You28] in order to model the response of the sustained acceleration. The resulting otoliths dynamical model has the following transfer function:

$$\frac{\hat{a}}{a} = 0.4\frac{0.06s + 1}{(1 + 5.33s)(1 + 0.66s)} \quad (6)$$

where $\hat{a}$ is the perceived acceleration and a is the acceleration of the driver's head.

Young and Oman [You29] modified the semicircular canals dynamical model that was proposed by Steinhausen [Ste25], by integrating additional time constant to include the adaptation effect of the sustained angular acceleration. The proposed model that reflects the response of the perception organs to the rotation movements has the following transfer function:

$$\frac{\hat{\omega}}{\omega} = 5.41\frac{30s^2}{(1 + 5.3s)(1 + 0.1s)(1 + 30s)} \quad (7)$$

where $\hat{\omega}$ is the perceived angular velocity and ω is the angular velocity of the driver's head.

Therefore, state space differential equations corresponding to the Eq.6 and Eq.7 can be written as:

$$\begin{aligned} \dot{x}_{oto}(t) &= A_{oto}x_{oto}(t) + B_{oto}u_{oto}(t) \\ y_{oto}(t) &= C_{oto}x_{oto}(t) + D_{oto}u_{oto}(t) \\ \dot{x}_{sc}(t) &= A_{sc}x_{sc}(t) + B_{sc}u_{sc}(t) \\ y_{sc}(t) &= C_{sc}x_{sc}(t) + D_{sc}u_{sc}(t) \end{aligned} \quad (8)$$

where A_{oto}, B_{oto}, C_{oto}, D_{oto}, A_{sc}, B_{sc}, C_{sc} and D_{sc} are matrices modeling the vestibular system (filter).

The vestibular system of the humans can only detect the movements if they are above the perception thresholds. Many studies in series of experiments reported that the human detection threshold of the rotational movements is between 0.1°/s and 3.0°/s and the detection threshold of the linear motions is between 0.014 m/s^2 and 0.25 m/s^2. These detection threshold values depend on the duration of motion stimuli as well as the rates of the acceleration and they vary from person to person [Ste25, Ben2, Ben3, Gue11, Gun12, and Zac30].

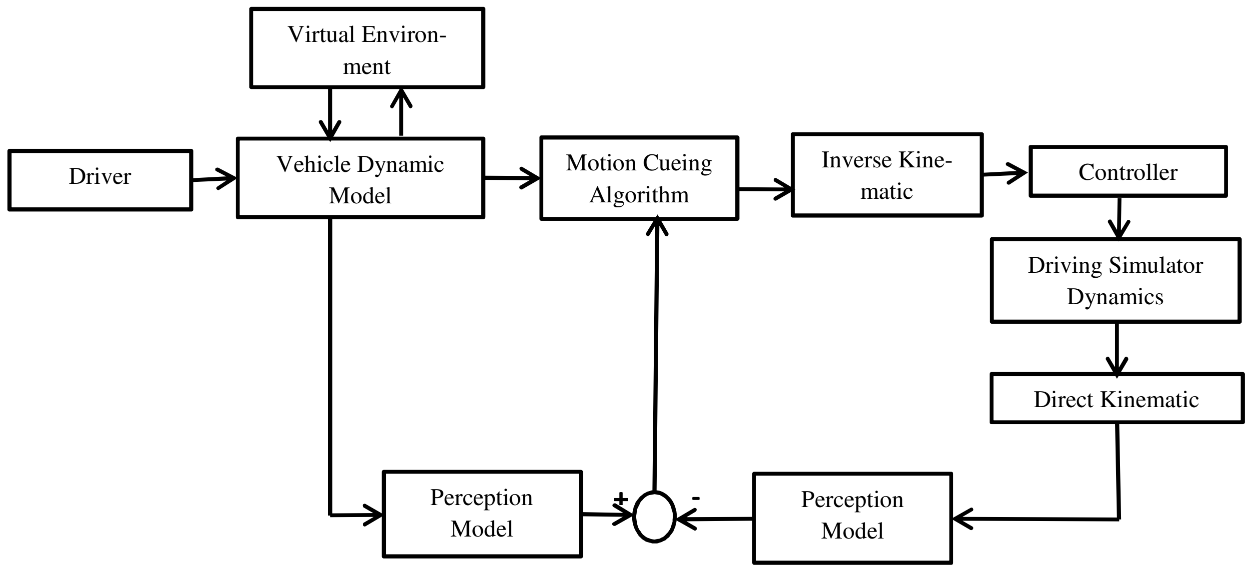

Figure 5: Motion Control Strategy

C. Constrained Linear Quadratic Optimal Controller

Figure (5) provides an overview of the block diagram of the proposed control strategy of the ATMOS driving simulator. The dynamic virtual reality model which consists of the vehicle dynamic model and the virtual environment generates according to the driver inputs, the movement trajectories which should be rendered by the motion cueing algorithm into restricted movements achievable by the simulator and at the same time gives the driver a realistic impression of the simulated vehicle motion.

The input signals of the driving simulators are the linear positions in longitudinal, lateral and vertical directions; the tilting angles resulting from the sustained longitudinal and lateral accelerations as well as the roll and pitch angles. The inverse kinematic modeling is used to determine the actuators angles by the given simulator middle point and the direct kinematic modeling is used to determine the middle point of the simulator by the given actuators angles.

In order to extract the acceleration that is used to calculate the tilt angle of the body of the driver around the x-axis and y-axis, the linear movement perception filter is used. By using this approach, many sensation dynamics missed by using the classical approaches are presented here and the rotation speed is done under the threshold of the perception model. In addition, the limitations on the extracted tilt coordination acceleration are guaranteed by using the proposed constrained linear quadratic controller.

The tilt angles can be calculated by [Rey20]:

$$\theta_{pitch} = sin^{-1}\left(\frac{a_{tilt-long.}}{g}\right), \theta_{roll} = sin^{-1}\left(\frac{a_{tilt-lateral}}{g}\right) \quad (9)$$

where θ_{pitch} and θ_{roll} are the tilt angles around y-axis and x-axis respectively, $a_{tilt-long.}$ and $a_{tilt-lateral}$ are the extracted accelerations form the vehicle signals that are used to generate the tilt angles, and g is the acceleration of the gravity. The tilt rotation speed $\frac{d\theta_{roll}}{dt}$ and $\frac{d\theta_{pitch}}{dt}$ should be limited to the perception threshold of the semicircular canals.

In this proposed motion control approach, the comparison is carried out between the perceived signals at the vehicle and that at the simulator. Therefore, $a_{tilt-long.}$ and $a_{tilt-lateral}$ are added to the extracted transient accelerations that are used for linear movements of the simulator in longitudinal direction (a_x) and in lateral direction (a_y). Therefore,

$$\begin{aligned} a_{s/x} &\cong a_x + a_{tilt-long.} \\ a_{s/y} &\cong a_y + a_{tilt-lateral} \end{aligned} \quad (10)$$

In order to simplify the notation, the matrices of the discrete state space model are denoted as in the continuous time models. The augmented discrete-time states $x_s = [x_m \quad x_{oto/sc} \quad x_{oto}]$ are the states of dynamic model of the simulator (x_m), the semicircular canals or otoliths dynamical model ($x_{oto/sc}$) and the states of the otoliths dynamical model (x_{oto}). Therefore, the augmented discrete-time state space differential equations can be defined as:

$$\begin{aligned} x_s(k+1) &= A_s x_s(k) + B_s u_s(k) \\ y_s(k) &= C_s x_s(k) + D_s u_s(k) \end{aligned} \quad (11)$$

where

$$A_s = \begin{bmatrix} A_m & 0 & 0 \\ B_{oto/sc}C_m & A_{oto/sc} & 0 \\ 0 & 0 & A_{oto} \end{bmatrix}, \; B_s = \begin{bmatrix} B_m & 0 \\ B_{oto/sc}D_m & 0 \\ 0 & B_{oto} \end{bmatrix}$$

$$C_s = \begin{bmatrix} D_{oto/sc}C_m & C_{oto/sc} & C_{oto} & 0 \\ 0 & 0 & 0 & C_m \end{bmatrix}, \; D_s = \begin{bmatrix} D_{oto/sc}D_m & 0 \\ D_m & D_{oto} \end{bmatrix}$$

where u_s are the control inputs of the simulator and y_s are the perceived signal at the simulator, the velocity and the position or angle of the simulator. A_s, B_s, C_s and D_s are matrices with appropriate dimensions.

In this strategy, an optimization problem is solved online over a finite time horizon (*N*). The objective of this optimization problem is to generate a control sequence that minimizes the difference between the perceived signals at the vehicle and the perceived signals at the simulator and keeps the simulator within its physical constraints and capabilities. The objective function that has to be minimized has the following form

$$J=\sum_{i=0}^{N-1}\left\|\left(\hat{a}_s(k+i|k)-\hat{r}(k+i|k)\right)\right\|_Q^2+\left\|\Delta u_s(k+i|k)\right\|_R^2 \quad (12)$$

subject to simulator constraints:

$$\begin{aligned}
&x_s(k+1+i|k)=A_s x_s(k+i|k)+B_s u_s(k+i|k), && i=0\cdots N-1\\
&y_s(k+i|k)=C_s x_s(k+i|k)+D_s u_s(k+i|k), && i=0\cdots N-1\\
&x_{min}\le x_s(k+1+i|k)\le x_{max}, && i=0\cdots N-1\\
&u_{min}\le u_s(k+i|k)\le u_{max}, && i=0\cdots N-1\\
&y_{min}\le y_s(k+i|k)\le y_{max}, && i=0\cdots N-1
\end{aligned} \quad (13)$$

where *J* is the cost function, $\hat{a}_s$ are the perceived reference signal at the simulator, the velocity and the position of the middle point, $\hat{r}$ is the perceived signal at the vehicle where $r(k)=\cdots=r(k+N-1)$, *R* and *Q* are positive definite weighting matrices, *k* is the computational step, and Δu_s are the input increments. Here the input increments are used instead of the input signals since in case of steady state, the input is not zero for offset free tracking but Δu_s is zero, therefore, by using Δu_s the cost function *J* is guaranteed to be consistent with zero tracking errors. Δu_s is defined as:

$$\Delta u_s(k)=u_s(k)-u_s(k-1) \quad (14)$$

In order to bring the simulator back to its neutral position, additional reference signals are added to the perceived reference signals. The neutral position is added as a reference position for the actual position of the simulator and a velocity with a value less than the perception threshold of the linear motion, is set as a reference signal for the simulator actual velocity. By this additional reference signals, the simulator washout can be ensured which allows more freedom of movements for the next movements and avoids actuators saturation.

Now the fixed horizon optimal control problem (Eq.12 and Eq.13) for linear systems with quadratic cost function and linear inequality and equality constraints can be set up as a quadratic program. In the following part, the state, output and control sequences will be reformulated for the finite time horizon *N* for each computational step *k*.

The recursive state and output sequences from 0 to *N-1* can be written as:

$$\begin{aligned}
X(k)&=\Psi_x x_s(k)+\Gamma_x u_s(k-1)+\Theta_x\Delta U(k)\\
Y&=\Psi x_s(k)+\Gamma u_s(k-1)+\Theta\Delta U(k)
\end{aligned} \quad (15)$$

where *X, Y* and $\Delta U(k)$ are the state, output and input increment sequences respectively, $\Psi_x,\Gamma_x,\Theta_x,\Psi,\Gamma$ and Θ are the recursive matrices of the state space model matrices A_s, B_s, C_s, and D_s. At steady state, where $\Delta u_s=0$ and the free tracking error $E(k)=0$, $R_{ref}=Y$. Therefore, the reference signal can be assumed as:

$$R_{ref}=\Psi x_s(k)+\Gamma u_s(k-1)+E(k) \quad (16)$$

where *E(k)* is the tracking error.

Then, the unconstrained optimal solution of the cost function is:

$$\Delta U_{opt}^{uc}=\frac{1}{2}(\Theta^T\tilde{Q}\Theta+\tilde{R})^{-1}\Theta^T\tilde{Q}E(k) \quad (17)$$

where $\tilde{Q}$ =blockdiag{*Q*,...,*Q*}, $\tilde{R}$ =blockdiag{*R*,…,*R*}.

Since $(\Theta^T\tilde{Q}\Theta+\tilde{R})^{-1}$ does not depend on the computational step *k*, it can be calculated offline.
From the last calculations, the equality constraints in the Eq.13 are eliminated by integrating them in the cost function. In order to handle the linear inequality constraints of the motion system, the inequality constraints can be represented as:

$$L\Delta u_s\le W \quad (18)$$

where *L* is a matrix which consists of identity matrices and a combination of the state space matrices of the dynamical model (Eq.11) and *W* is a vector of all constrains of the motion system define in Eq.13. In *W* vector, the constraints on the output and the states are defined in terms of the last input control signal, the current estimation of the state vector and the limits of the motion system, therefore *W* vector should be calculated at each computational step, whereas *L* can be calculated offline.

Using the above formulation, the optimal solution $\Delta U(k)_{opt}$ of minimizing Eq.12 subject to the inequality constraints in Eq.13 can be expressed as a quadratic problem:

$$\begin{aligned}
\Delta U(k)_{opt}=&\underset{L\Delta u\le W}{\arg}\ \underset{\Delta U}{\min}\Delta U(k)^T(\Theta^T\tilde{Q}\Theta+\tilde{R})\Delta U(k)\\
&-2\Delta U(k)^T\Theta^T\tilde{Q}E(k)+E(k)^T\tilde{Q}E(k)
\end{aligned} \quad (19)$$

The matrix $(\Theta^T\tilde{Q}\Theta+\tilde{R})$ is called the Hessian of the quadratic problem. Since $\tilde{Q}$, $\tilde{R}$ and the Hessian matrix are positive definite, the quadratic programming problem is convex.

Many standard numerical quadratic programming algorithms are available to solve the above optimization problem such as the active set method [Goo9, Fle8] or interior point methods [Fle8, Wri27].

Since the controller is supposed to run continuously, a common way to apply the linear quadratic optimal controller is by using only the first *m* rows of the calculated optimal controller, therefore

$$\Delta u_s(k)_{opt} = \lfloor I_m \quad 0_m \quad \ldots \quad 0_m \rfloor \Delta U(k)_{opt} \qquad (20)$$

The designed controller here has a feedback from the measured accelerations of the simulator and has a feedforward part from the reference trajectory.

Simulation Results

Since the ATMOS driving simulator is fully actuated, the control of each degree of freedom will be carried out independently but synchronously. ATMOS driving simulator features a unique moving platform structure. Basically, this moving platform features unique motion capabilities in terms of combined motion i.e. when the simulator moves in longitudinal or lateral direction, it has the ability to rotate (tilt) around the y-axis or x-axis respectively with tilting angular velocity under the perception threshold of the semicircular canals. Therefore, a major part of the sustained acceleration will be rendered directly through the movements of the moving platform and the other part will be rendered through tilting the shaker system i.e. adding the resulting tilt angle to the corresponding angle generated by rendering the roll and pitch velocities of the vehicle.

In this section, simulation results show the performance comparison between the classical motion cueing algorithm and the proposed motion control method. The Automotive simulation model(ASM-dSPACE) is used to generate the accelerations of the vehicle. A scenario of normal driving situation consisting of an assortment of accelerations, decelerations and braking maneuvers is carried out. The generated translational accelerations and angular velocities by ASM are used as reference inputs for the motion control strategies. The simulations are executed by using Matlab/Simulink. In this part, simulation results only show the rendering of the longitudinal acceleration and the pitch velocity and the remaining degree of freedoms are rendered in a similar manner. The parameters of the classical motion algorithm are adjusted to keep the simulator within its constraints and the positive-definite weighting matrices *R* and *Q* are chosen by trial and error to guarantee the best tracking of the perceived signals at the vehicle. Table 1 shows the capabilities of ATMOS driving simulator and figures 6 and 7 show the workspace of the shaker and moving platform respectively.

Table (1) ATMOS driving simulator capabilities

	Displ.\Rotation	Velocity	Acceleration
Longitudinal	±733 (mm)/ ±13.5°	±1.3 m/s	±3 (m/s^2)
Lateral	±522 (mm)/ ±10°	±1 m/s	±2.2 (m/s^2)
Vertical	±72.5 (mm)	±0.6 m/s	±4 (m/s^2)
Pitch	±7 (°)	±50 (°/s)	±360 (°/s^2)
Roll	±7 (°)	±50 (°/s)	±360 (°/s^2)

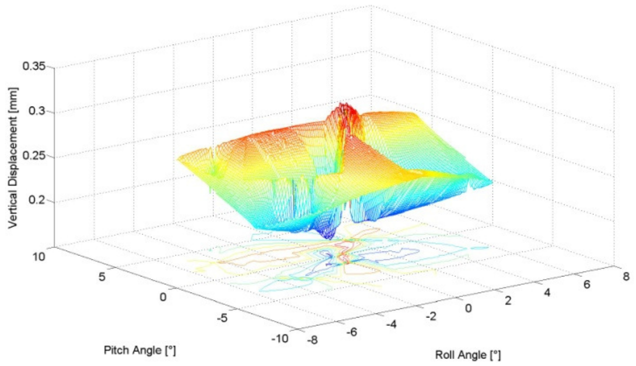

Figure 6: Shaker workspace

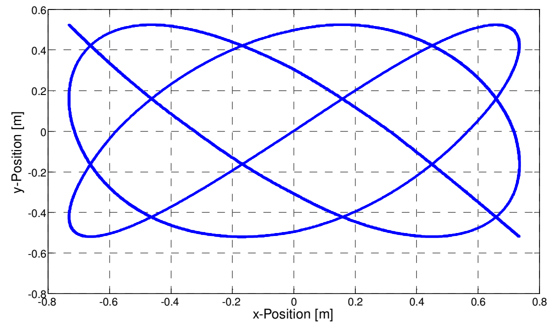

Figure 7: Moving platform workspace

In order to validate the modeling of ATMOS driving simulator such as in longitudinal direction, a sinus signal with different amplitude is applied to the two linear actuators of the simulator. Figure 8 shows a good matching between the measured and modeled longitudinal acceleration.

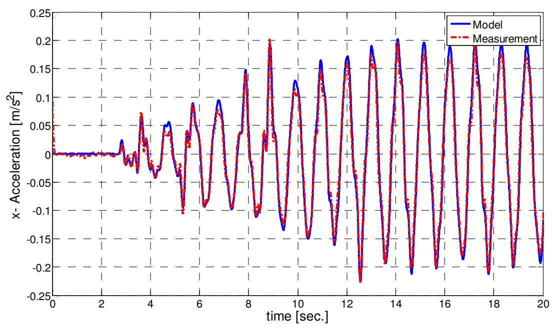

Figure 8: Acceleration in x- direction

Figure 9 shows a comparison between the transient acceleration of the platform and the vehicle longitudinal acceleration. However, Figures 10 and 11 show a comparison between the perceived signals at the simulator generated by the classical motion cueing algorithm and by the new approach as well as the perceived signal at the vehicle. The results show a very good matching between the two perceived signals at the driving simulator generated by the new approach and at the vehicle.

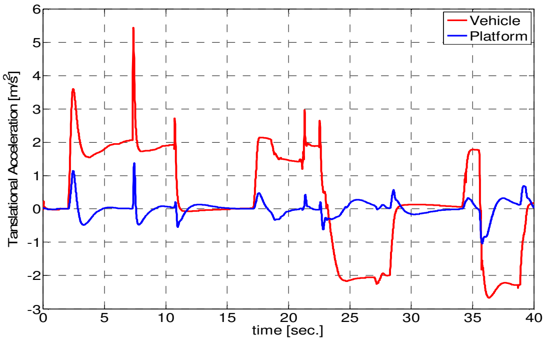

Figure 9: Vehicle and platform translational acceleration

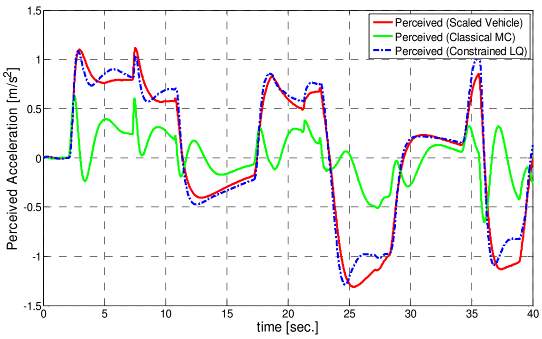

Figure 10: Perceived longitudinal acceleration

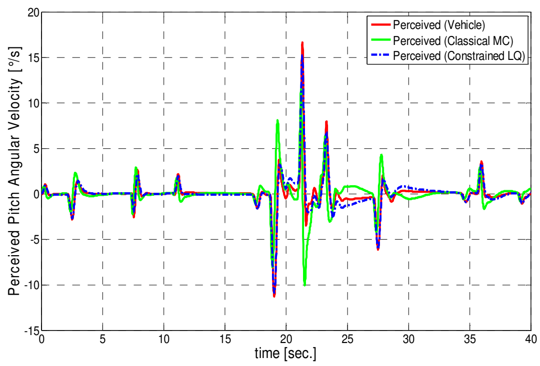

Figure 11: Perceived pitch angular velocity

Figure 12 shows the tilting angular rate generated by the new approach. It is clear that the tilting velocity is guaranteed to be under the detectable thresholds of the perception system of the driver.

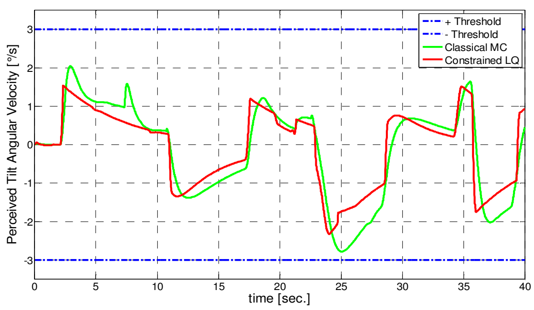

Figure 12: Tilting angular rate

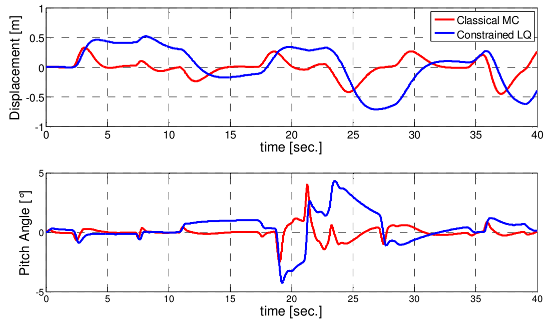

Figure 13: Longitudinal displacement and pitch angle

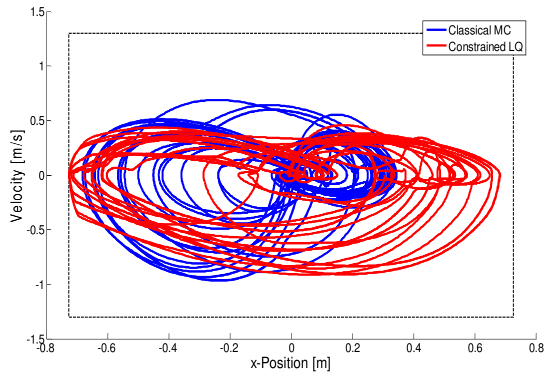

Figure 14: Exploited workspace during 5 minutes of simulated driving session

Conclusion

The purpose of the motion cueing algorithms is to render the physical movements of the vehicle so that the driver of the simulator perceives the motion in virtual environment similar to reality. The new motion control strategy based on the constrained linear quadratic optimal controller was developed with a focus on reducing the perception difference between the perceived signal at the simulator and the perceived signal at the vehicle. Therefore, the proposed optimal controller provides the simulator with sequences of control signals that minimize the difference between the perceived acceleration at the vehicle and the perceived acceleration at the simulator and take into account the simulator constraints and capabilities.

A case study was conducted by two trajectories to compare the performance of the proposed model-based approach with the classical motion cueing algorithm. From the simulation results, it can be stated that the proposed approach provides more realistic impression than the conventional classical motion cueing and the exploited workspace is much better. In addition, the constraints of the simulator are always respected.

In future, the new constrained linear quadratic optimal controller will be implemented on the ATMOS driving simulator and a subjective quality criterion will be carried out to compare this new approach with the conventional well know approaches.

Acknowledgment

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